

Plasticity and Deformation Processes

Plastic deformation exercises

- An aluminum rectangular prism, with sides 60x80x100 cm long and that are oriented parallel to the the x, y, z axes, is subjected to normal forces in three dimensions with $F_x= 80000$ kN, $F_y= -12000$ kN, $F_z= -24000$ kN. The material undergoes plastic deformation as its yield strength is 30 MPa and $E= 70$ GPa, $\nu= 0.35$. The stress-strain curve of aluminum is approximated by the power law model with the equation $\sigma=500*\epsilon^{0.5}$ as given in the following figure. Determine the plastic deformation of the material on the three surface planes when the load is removed.

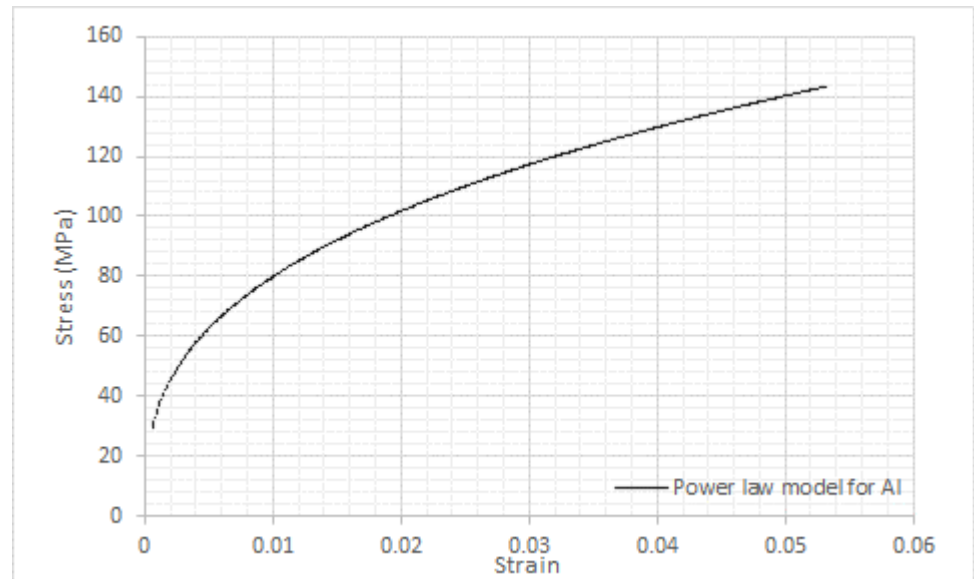
- $\sigma = A\epsilon^n \quad \sigma \geq \sigma_y$

- $E_{sec} = \frac{\sigma_y + A\epsilon^n}{\epsilon_T}$

- $\epsilon_T = \sqrt[n]{\left(\frac{\sigma_{eff} - \sigma_y}{A}\right)} + \epsilon_y$

- $\nu = \frac{1}{2} - \frac{E_{sec}}{E} \left(\frac{1}{2} - \nu^e\right)$

- $G^p = \frac{E_{sec}}{2(1+\nu)}$



- The aluminum rectangular prism is heated to 400 C, and is subjected to normal forces in three dimensions with $F_x = 80000$ kN, $F_y = -12000$ kN, $F_z = -24000$ kN. The material undergoes plastic deformation as its yield strength is 10 MPa and $E = 10$ GPa, $\nu = 0.45$. The stress-strain curve of heated aluminum is approximated by the power law model with the equation $\sigma = 14 \cdot \epsilon^{0.05}$ as given in the following figure. Determine the plastic deformation of the material on the three surface planes when the load is removed.

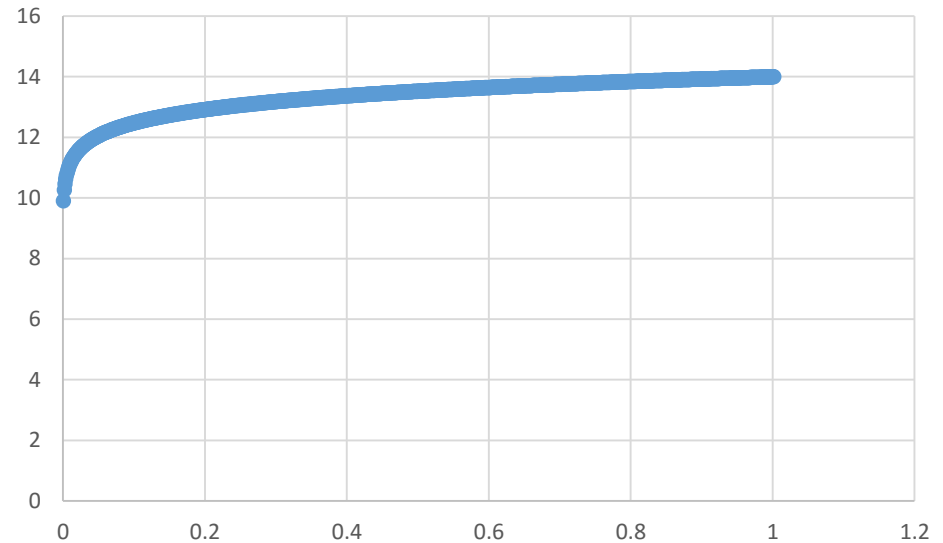
- $\sigma = A\epsilon^n \quad \sigma \geq \sigma_y$

- $E_{sec} = \frac{\sigma_y + A\epsilon^n}{\epsilon_T}$

- $\epsilon_T = \sqrt[n]{\left(\frac{\sigma_{eff} - \sigma_y}{A}\right)} + \epsilon_y$

- $\nu = \frac{1}{2} - \frac{E_{sec}}{E} \left(\frac{1}{2} - \nu^e\right)$

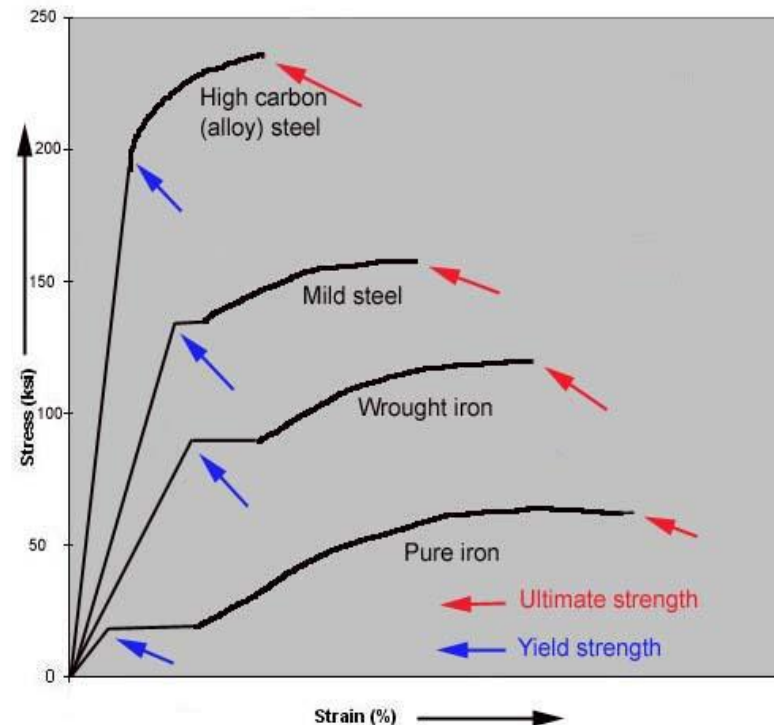
- $G^p = \frac{E_{sec}}{2(1+\nu)}$



- A mild steel cube, with sides 100x100x100 cm long is subjected to press forging with normal forces in the x and z directions, $F_x = -80000$ kN, $F_z = -100000$ kN. The yield strength of the material is 250 MPa and $E = 200$ GPa, $\nu = 0.29$. The stress-strain curve of mild steel is given below and can be approximated by the linear hardening model with the equation $\sigma = 248 + 1000\varepsilon$. Determine the deformation of the material on the three surface planes when the load is removed.

- $\sigma = \sigma_1 + E_1 \varepsilon \quad \sigma \geq \sigma_y$
- $E_{sec} = \frac{\sigma_1}{\varepsilon} + E_1$
- $\sigma_y = \sigma_1 + E_1 \varepsilon_y$
- $\nu = \frac{1}{2} - \frac{E_{sec}}{E} \left(\frac{1}{2} - \nu^e \right)$
- $G^p = \frac{E_{sec}}{2(1+\nu)}$

Comparative Stress/Strain Diagram



- The mild steel cube is heated to 800 C and is subjected to press forging with normal forces in the x and z directions, $F_x = -80000$ kN, $F_z = -100000$ kN. The material undergoes plastic deformation as its yield strength is 20 MPa and $E = 20$ GPa, $\nu = 0.44$. The stress-strain curve of mild steel is given below and can be approximated by the linear hardening model with the equation $\sigma = 19.9 + 50\varepsilon$. Determine the plastic deformation of the material on the three surface planes when the load is removed.

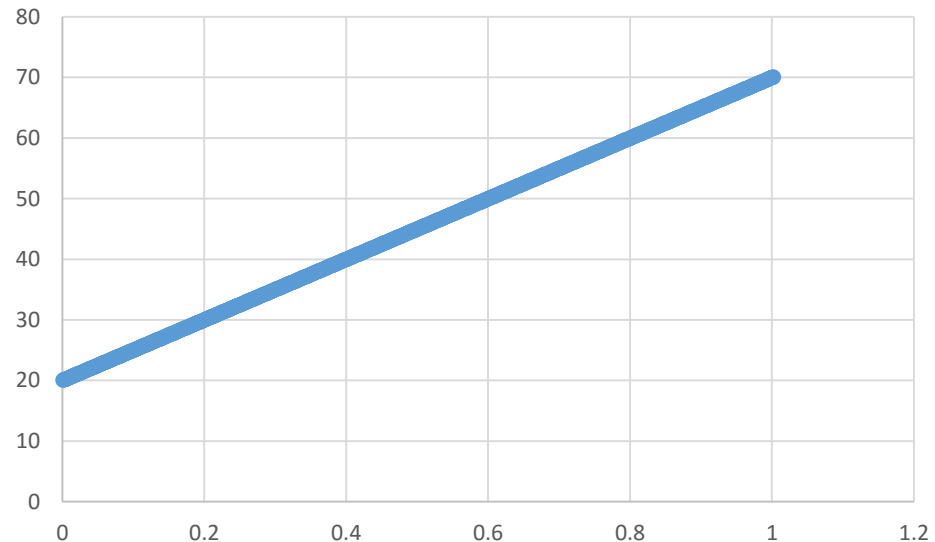
- $\sigma = \sigma_1 + E_1 \varepsilon \quad \sigma \geq \sigma_y$

- $E_{sec} = \frac{\sigma_1}{\varepsilon} + E_1$

- $\sigma_y = \sigma_1 + E_1 \varepsilon_y$

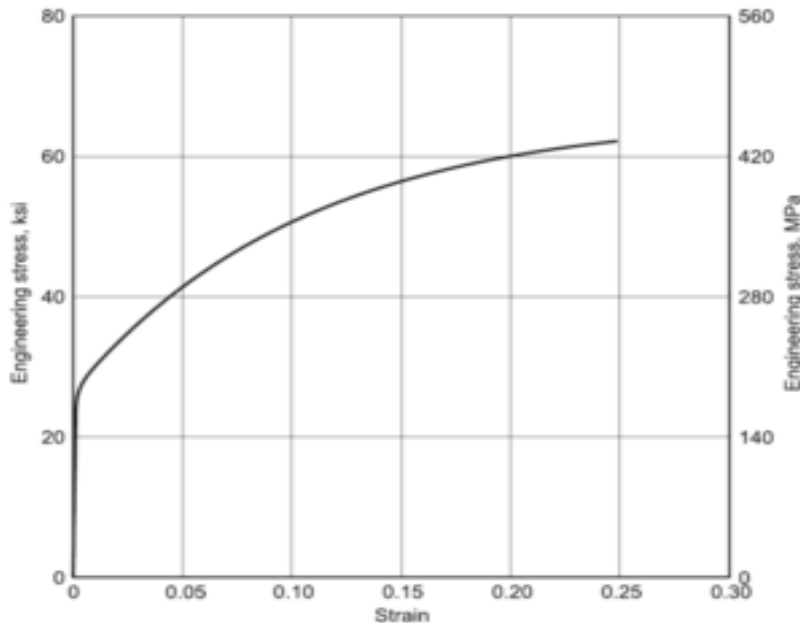
- $\nu = \frac{1}{2} - \frac{E_{sec}}{E} \left(\frac{1}{2} - \nu^e \right)$

- $G^p = \frac{E_{sec}}{2(1+\nu)}$



- A pure nickel rectangular slab, with sides 200x50x10 cm is subjected to rolling to produce sheet metal. The rollers apply friction to an area of 10 cm length and 50 cm width per minute. This friction creates a shear force at the zx plane of about 50000 kN. Determine if the material undergoes plastic deformation. Its yield strength is 185 MPa and $E = 207 \text{ GPa}$, $\nu = 0.31$. The stress-strain curve of pure nickel is given below and can be approximated by the power model with the equation $\sigma = 500 * \epsilon^{0.16}$. Calculate the total shear strain of the material as it is passed between rollers completely in 20 minutes.

Nickel (Ni)



Ni.001 Ni 200 annealed nickel sheet, engineering stress-strain curve (full range)

Test direction: longitudinal. Sheet thickness = 0.787 mm (0.031 in.). Commercially pure nickel (UNS N02200). 0.2% yield strength = 185 MPa (26.9 ksi); ultimate tensile strength = 434 MPa (63.0 ksi); elongation = 39.5%; strength coefficient (K) = 138.2; strain-hardening exponent (n) = 0.387. Composition: Ni 99.0 min

Courtesy of Special Metals Corporation

- A pure nickel rectangular slab, with sides 200x50x10 cm is subjected to rolling at 1000 C to produce sheet metal. The rollers apply friction to an area of 10 cm length and 50 cm width per minute. This friction creates a shear force at the zx plane of about 10000 kN. Determine if the material undergoes plastic deformation. Its yield strength is 15 MPa and $E= 20$ GPa, $\nu= 0.43$ at 1000 C . The stress-strain curve of pure nickel at 1000 C is given below and can be approximated by the power model with the equation $\sigma = 17 * \epsilon^{0.02}$. Calculate the total shear strain of the material as it is passed between rollers completely in 20 minutes.

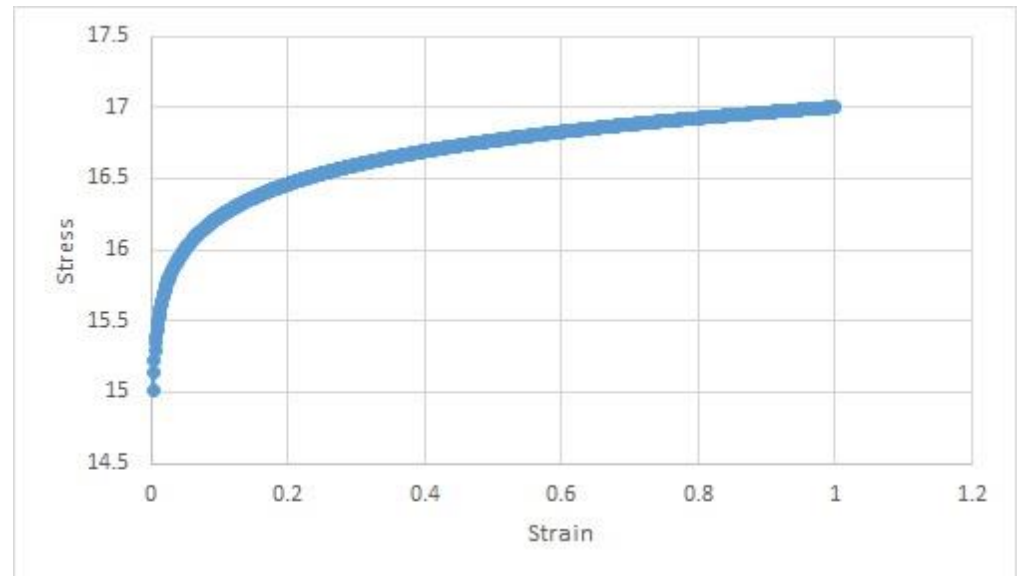
- $\sigma = A\epsilon^n \quad \sigma \geq \sigma_y$

- $E_{sec} = \frac{\sigma_y + A\epsilon^n}{\epsilon_T}$

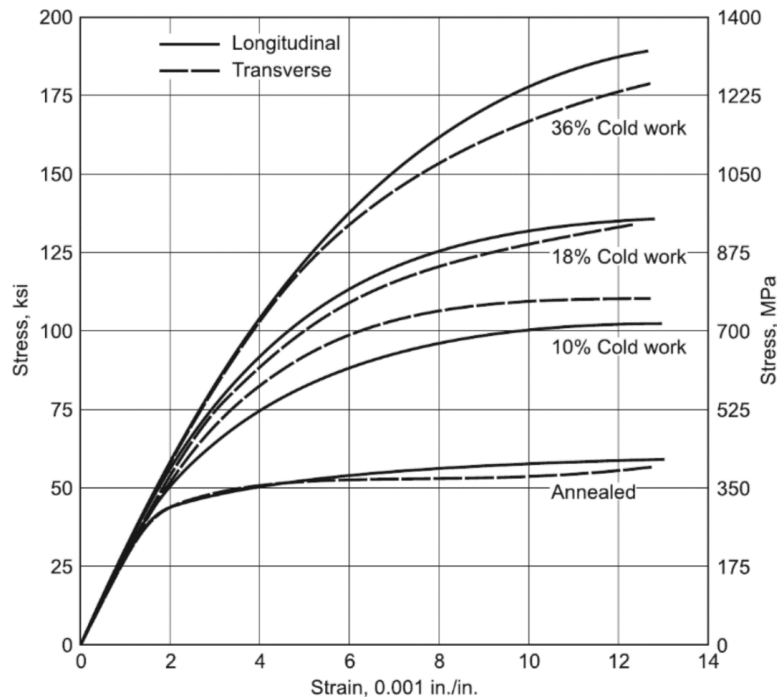
- $\epsilon_T = \sqrt[n]{\left(\frac{\sigma_{eff} - \sigma_y}{A}\right)} + \epsilon_y$

- $\nu = \frac{1}{2} - \frac{E_{sec}}{E} \left(\frac{1}{2} - \nu^e\right)$

- $G^p = \frac{E_{sec}}{2(1+\nu)}$



- A stainless steel blank disk, with diameter of 1 meter and thickness 1 cm is subjected to deep drawing to produce pressure cooker cups. The punch applies downward tensile force of 160000 kN on a circular area with diameter 50 cm and an average shear stress of 80 MPa on the cup walls. Determine if the material undergoes plastic deformation. Its yield strength is 300 MPa and $E = 200 \text{ GPa}$, $\nu = 0.3$. The stress-strain curve of stainless steel is given below and can be approximated by the linear hardening model with the equation $\sigma = 287 + 7500\varepsilon$. It can also be approximated by power law. Curve fit the stress-strain curve using a general equation of the form $\sigma = A\varepsilon^n$

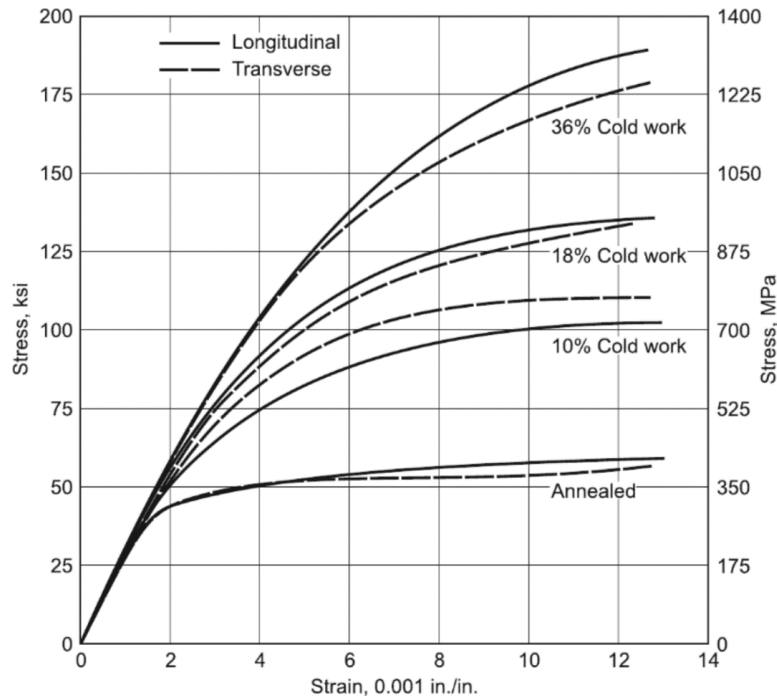


SS.001 201 stainless steel, stress-strain curves showing effect of cold work

Test direction: longitudinal and transverse. Composition: Fe-17Cr-6.5Mn-4.5Ni. UNS S20100

Source: P.D. Harvey, *Engineering Properties of Steel*, American Society for Metals, 1982

- A stainless steel blank disk, with diameter of 1 meter and thickness 1 cm is subjected to deep drawing to produce pressure cooker cups. The punch applies downward tensile force of 160000 kN on a circular area with diameter 50 cm and an average shear stress of 80 MPa on the cup walls. Determine if the material undergoes plastic deformation. Its yield strength is 300 MPa and $E = 200 \text{ GPa}$, $\nu = 0.3$. The stress-strain curve of stainless steel is given below and can be approximated by the linear hardening model with the equation $\sigma = 287 + 7500\varepsilon$. Calculate the normal and shear deformation of the material as it is punched down.



SS.001 201 stainless steel, stress-strain curves showing effect of cold work

Test direction: longitudinal and transverse. Composition: Fe-17Cr-6.5Mn-4.5Ni. UNS S20100

Source: P.D. Harvey, *Engineering Properties of Steel*, American Society for Metals, 1982

$$\sigma = \sigma_1 + E_1 \varepsilon \quad \sigma \geq \sigma_y$$

$$E_{sec} = \frac{\sigma_1}{\varepsilon} + E_1$$

$$\sigma_y = \sigma_1 + E_1 \varepsilon_y$$

$$\nu = \frac{1}{2} - \frac{E_{sec}}{E} \left(\frac{1}{2} - \nu^e \right)$$

$$G^p = \frac{E_{sec}}{2(1 + \nu)}$$

- A stainless steel blank disk, with diameter of 1 meter and thickness 1 cm is subjected to deep drawing at around 850 C to produce pressure cooker cups. The punch applies downward tensile force of 160000 kN on a circular area with diameter 50 cm and an average shear stress of 80 MPa on the cup walls. Determine if the material undergoes plastic deformation. Its yield strength is 55.5 MPa and $E = 15 \text{ GPa}$, $\nu = 0.4$. The stress-strain curve of stainless steel at 850 C can be approximated by the power law model with the equation $\sigma = 57\varepsilon^{0.005}$. Calculate the normal and shear deformation of the material as it is punched down.

- $\sigma = A\varepsilon^n \quad \sigma \geq \sigma_y$

- $E_{sec} = \frac{\sigma_y + A\varepsilon^n}{\varepsilon_T}$

- $\varepsilon_T = \sqrt[n]{\left(\frac{\sigma_{eff} - \sigma_y}{A}\right)} + \varepsilon_y$

- $\nu = \frac{1}{2} - \frac{E_{sec}}{E} \left(\frac{1}{2} - \nu^e\right)$

- $G^p = \frac{E_{sec}}{2(1+\nu)}$

